

## CONFIDENCE INTERVALS FOR CRONBACH'S RELIABILITY ALPHA COEFFICIENT

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### ABSTRACT

Cronbach's reliability alpha coefficient (Cronbach, 1951) was suggested as a measure of a questionnaire's internal consistency. The term reliability though is due to Spearman (1904), who first said that errors exist even in the non sampling cases. He noted that these errors can be estimated by the size of consecutive and repetitive measures. Each measurement consists of two elements, the true value of the measurement and the error of the measurement. If the measurement is repeated it will yield new values both for the measurement and the error associated with it. The reliability is actually the ratio of the true value divided by the observed (through a measurement) value.

The aim of this paper is to investigate interval estimation of the alpha coefficient. Maydeu-Olivares and Coffman (2007) provided an asymptotic distribution-free interval estimation of the alpha coefficient. Koning and Franses (2006) suggested a transformation for constructing confidence intervals for the true value of alpha. Padilla et al. examined confidence intervals for the alpha coefficient using bootstrap with non normal populations.

We provide an alternative transformation of the alpha coefficient and construct confidence intervals for the true value. In order to do this we employ the asymptotic distribution of the estimate of the coefficient van Zyl et al. (2000). Simulation studies are performed to demonstrate our methodology when the data are continuous and when they are converted to a Likert scale. A comparison with some present methodologies including the bootstrap methodology is presented as well.

Keywords: reliability, Cronbach's alpha, transformation, asymptotic normal, delta method, confidence intervals

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