

LOOKING FOR A FLEXIBLE FUNCTIONAL FORM: THEORY AND EVIDENCE

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Abstract

This paper investigates the econometric application of a flexible functional form of Generalised Leontief function. We will focus our attention on econometric techniques that are used to analyze the interrelated demands for labour, capital and other inputs in production processes. In particular, we begin with an historical overview of the literature on the demand for factors of production, beginning with the famous Cobb-Douglas model and constant elasticity of substitution (CES) and moving with the application of Generalised Leontief function using data from the Greek industry.

1. INTRODUCTION

The notion of a production function has been used for a long time. In the theoretical literature, in the first edition of famous text in 1890 *Principles of Economy*, Alfred Marshall has emphasized on theoretical relationships between production function and factor demands, however, the empirical analysis lagged considerably behind theoretical developments.

Paul Douglas was very devoted considerable attention in explaining the movements of labour productivity and real wages over time.¹ Douglas wanted to test the marginal productivity theory. The important issue for him was if labour was in fact paid the value of its marginal product. Cobb and Douglas assumed that production was characterized by constant returns to scale.

They related empirically in a logarithmic form the value added output to the inputs of capital and labour for the U.S. manufacturing based on annual data for the period 1899-1922.

$$\ln Y = \ln A + \hat{\alpha}_K \ln K + \hat{\alpha}_L \ln L$$

where: Y are the output (value added), and K, L is capital and labour respectively .

The assumption of constant returns to scale (or otherwise the homogeneity of degree one) implied the restriction for the parameters $\hat{\alpha}_K + \hat{\alpha}_L = 1$.² Rearranging the above logarithmic equation of labour productivity to the capital/labour ratio:

$$\ln(Y/L) = \ln A + \hat{\alpha}_K \ln(K/L)$$

The corresponding nonlogarithmic form with constant returns to scale which using for the empirical implementation has the following form:

$$Y = A K^{\hat{\alpha}} L^{1-\hat{\alpha}}$$

Rearranging the above nonlogarithmic equation and taking the partial derivatives of Y with respect to K and L and equating the marginal products with the real input prices and solving, we can obtain:

$$\alpha_K = \frac{P_K K}{PY}$$

and

¹ Cobb Charles and Paul H. Douglas (1928), "A Theory of Production", American Economic Review, Supplement Vol. 18, pp. 139-165.

² The nonlogarithmic form is $Y = A K^{\hat{\alpha}} L^{\hat{\alpha}}$, and multiplying by $\hat{\alpha} > 1$, we have: $\hat{\alpha} Y = A (\hat{\alpha} K^{\hat{\alpha}})(\hat{\alpha} L^{\hat{\alpha}}) = A K^{\hat{\alpha}} L^{\hat{\alpha}} \hat{\alpha}^{\hat{\alpha}+1}$. This function is homogeneous of degree $\hat{\alpha} + \hat{\alpha} = 2\hat{\alpha}$ and when $\hat{\alpha} = 1$ then $\hat{\alpha} + \hat{\alpha} = 1$.

$$\alpha_L = 1 - \alpha_K = \frac{P_L L}{PY}$$

Cobb-Douglas argued that if markets were competitive, if marginal products equated to the real prices and if production technology following the constant returns to scale, then the least squares estimates of the parameters $\hat{\alpha}_E$ and $\hat{\alpha}_L$ should be equal approximately to the value shares of capital and labour.

Other economists were more interested in measuring substitution elasticities among inputs. They defined the substitution elasticity between the capital and labour as following:

$$\sigma = \frac{\partial \ln(K/L)}{\partial \ln(F_L/F_K)} = \frac{\partial \ln(K/L)}{\partial \ln(P_L/P_K)}$$

where: F_K and F_L are the marginal products of capital and labour respectively.

For the Cobb-Douglas function the substitution of elasticity (σ) always equals to unity. The first empirical paper attempted to measure the substitution elasticities between the inputs using the theory of cost and production was the Ragnar Frisch, who estimated a substitution coefficient (the ratio of marginal productivities) between the inputs.³

Later, an extension of the Cobb-Douglas function introduced by Kenneth Arrow, Hollis Chenery, Bagicha Minhas, and Robert Solow; in their model they tried to search in which functional form the substitution of elasticity (σ) will be constant but not constrained to unity. They concluded in the following equation:

$$\ln(K/L) = \text{constant} + \sigma \ln(F_K/F_L)$$

where the second term (F_K/F_L) indicates the marginal rate of substitution.

The above function indicates the well-known constant elasticity of substitution (CES) production function with constant returns to scale can be written as following:

$$Y = A[\delta K^{-\rho} + (1 - \delta)L^{-\rho}]^{-1/\rho}$$

where the substitution of elasticity $\sigma = 1/(1+\rho)$ ⁴

In fact, the constant elasticity of substitution (CES) production function has appeared in the literature a quarter century earlier than Cobb-Douglas production function. The constant elasticity of substitution (CES) production function has been derived on consumer demand analysis. Abraham Bergson⁵ used the following function:

$$Y^{-\rho} = A \left(\sum_{i=1}^n \delta_i X_i^{-\rho} \right)$$

Nerlove estimated a three input Cobb-Douglas cost function (namely, capital, labour and fuels) with returns to scale to be other than constant; his empirical analysis indicated that the returns to scale were increasing rather than constant.⁶ However, Nerlove was unsatisfied with the restricted assumptions of Cobb-Douglas function according to which the substitution of elasticities required to be equal to unity and by constant elasticity of substitution (CES) which implied some other restrictions for the substitution of elasticities that required to be constant and equal to each other.

³ Frisch Ragnar (1935) "The principle of substitution: an example of its application in the chocolate industry", *Nordisk Tidsskrift for Teknisk Okonomi*, 1:1, pp: 12-27.

⁴ There is a limiting case in which $\rho \rightarrow 0$, $\sigma \rightarrow 0$, the Cobb-Douglas function is a limiting form of the CES function.

⁵ Bergson (Burk) Abraham (1936) "Real Income, Expenditure proportionality, and Frisch's New Method of Measuring utility", *Review of Economic Studies*, vol. 4:1, October, pp: 33-52.

⁶ Nerlove Marc (1963) "Returns to Scale in Elasticity Supply" in Carl Christ ed. *Measurement in Economics: Studies in Mathematical Economics and Econometrics in Memory of Yehuda Grunfeld*, Stanford, California, Stanford University Press, pp: 167-198.

Lucas et al. have attempted to reconcile the seemingly disparate cross-sectional and time-series estimates of substitution of elasticity (σ).⁷

In 1961, Earl Heady and John Dillon in their book of *Agricultural Production Function*⁸ experimented with Taylor's series expansion introduced the second-degree polynomial in logarithms that added quadratic and cross-terms to the Cobb-Douglas function. They estimated the production function directly using least square methods and called this procedure production function contour fitting. They reported the least squares estimates of a square root transformation that included as a special case the generalized linear production function that introduced in 1971 by Erwin Diewert.⁹ Diewert's Generalized Leontief Functional Form was the first in the theory of dual cost and production.

Daniel McFadden¹⁰ focused on the theory and its applications of duality in production. He examined both the use of duality theory and the problem of generating more flexible functional forms with more than two or three inputs and less restrictive forms than the Cobb-Douglas and constant elasticity of substitution (CES) specifications.

Several other empirical results have been reported in the literature. In 1971, Nerlove have been surveyed empirical findings¹¹, and in 1973 Berndt has been summarized additional empirical findings.¹²

A decade later, in 1970 Laurits Christensen, Dale W. Jorgenson, and Lawrence J. Lau introduced a flexible functional form the "translog production function" a form that placed no-restrictions on the substitution of elasticities.¹³ The translog function was a second order Taylor's series in logarithms and was identical to the production function considered by Heady some decades earlier.¹⁴

Lastly, the Boskin and Lau introduced another flexible functional form "the meta-production function" that is an extension of translog production function and can be employed with the panel or pool data.¹⁵ This function form places no a priori restrictions on the substitution possibilities among the inputs of production. It also allows scale economies to vary with the level of output. This feature is essential to enable the unit cost curve to attain the classical shape.

Econometric applications of cost and production functions differ in their assumptions. In the regression of the production function, output is endogenous and input quantities are exogenous. In the dual cost function, the production costs and the input quantities are endogenous. When output and input prices can be considered as exogenous, then it is better to apply a cost function that has input prices as regressors, rather than a production function in which input quantities are the right-hand variables.¹⁶

Empirical research on estimating cost and production function relationships has a long history, trying to explain the average labour productivity, the relationships between inputs and outputs, to estimate the substitution elasticities among inputs and finally to estimate the returns to scale.

The production function parameters can be uniquely recovered from estimation of the demand equations derived from the dual cost function.¹⁷

The empirical analysis of input demands and input substitution patterns provides an example of the strong links between economic theory and econometric implementation. The econometric techniques that we employ deal with estimation of parameters in systems of equations.

Furthermore, the implementation of a multiproduct cost functions can permit a richer analysis of the effects on costs and factor demands of various changes in the composition and levels of output; some recent

⁷Lucas Robert (1969) "Labour-Capital Substitution in U.S. Manufacturing" in Arnold C. Harberger and Martin J. Bailey eds. *The Taxation of Income from Capital*, Washington, D.C.: The Brookings Institution, pp: 223-274.

⁸Heady Earl and John L. Dillon (1961) *Agricultural Production Functions*, Ames, Iowa: Iowa State University Press.

⁹Diewert Erwin (1971) "An Application of the Shepard Duality Theorem: A Generalized Linear Production Function", *Journal of Political Economy*, vol. 79:3, May/June, pp: 482-507.

¹⁰McFadden Daniel (1978) *Production Economics: A Dual Approach to Theory and Applications*, volume 1, Amsterdam: North-Holland

¹¹Nerlove Marc (1967) "Recent empirical studies of the CES and Related Production Functions" in Murray Brown ed., *The Theory and Empirical Analysis of Production*, Studies in Income and Wealth, vol. 32, New York, Columbia University Press for the National Bureau of Economic Research, pp: 55-122.

¹²Berndt Ernst (1976) "Reconciling Alternative Estimates of the Elasticity of Substitution", *Review of Economics and Statistics*, vol. 58:1, February, pp: 59-68.

¹³Christensen Laurits, Dale W. Jorgenson and Lawrence J. Lau (1971) "Conjugate Duality and the Transcendental Logarithmic Production Function", *Econometrica*, vol.39:4, July, pp: 255-256, and Christensen Laurits, Dale W. Jorgenson and Lawrence J. Lau (1973) "Conjugate Duality and the Transcendental Logarithmic Production Function", *Review of Economics and Statistics*, vol. 55:1, February, pp: 28-45.

¹⁴Heady, however, has emphasized only on the primal production function and he did not consider the dual cost or even the specifications of the profit function.

¹⁵Boskin M.J. and Lau L.J.: (1992) "Capital, technology and Economic growth", chapter 2 in Rosenberg, Landau and Mowery (ed.) *Technology and the wealth of nations*, Stanford University press.

¹⁶Zellner Arnold, Jan Kmenta and Jaques Dreze (1966) "Specification and estimation of Cobb-Douglas Production Function Models", *Econometrica*, vol.34:3, October, pp: 784-795.

¹⁷A cost function is dual in the sense that it embodies all the parameters of the underlying production function. See Berndt Ernst (1991) *The Practice of Econometrics: Class and Contemporary*, Addison-Wesley Publishing Company.

examples of empirical implementation of multiproduct cost functions can be found among others in Douglas Caves and Lauritis Christensen.

2. THE THEORETICAL FRAMEWORK

There are a number of ways to approach the estimation of production function and technical progress. The aim of this section is to examine the nature of technological progress and factor substitution in the Greek industry by using the *Generalised Leontief production function* for the annual time-series data in the period 1959-1990.

The *Generalised Leontief functional form* which proposed by Diewert has been established as a useful alternative for the long-run production studies.

We can consider the following *Generalised Leontief functional form* for a cost function¹⁸ with the constant returns to scale that can be written as:

$$C = Y \left[\sum_{i=1}^n \sum_{j=1}^n \gamma_{ij} (w_i w_j)^{1/2} \right] \quad (1)$$

where: C is the total cost, Y is the output, w_{ij} the prices of n inputs ($i, j = 1, \dots, n$), and Q_{ij} the n input quantities, with $\square_{ij} = \square_{ji}$ ($i, j = 1, 2, \dots, n$). The parameters $\tilde{a}_{ij}$ are such that (1) $\tilde{a}_{ij} = \tilde{a}_{ji}$ and (2) $\tilde{a}_{ij} \geq 0$ (for $i, j = 1, 2, \dots, n$).

Let us assume that we have n inputs, as w_i , ($i = 1, \dots, n$), with the n input quantities Q_i , and the total cost indicating by C and the output by Y. We are assuming that the output and the input prices Y and w_{ij} are exogenous, while the input quantities Q_{ij} are endogenous.

The cost C which defined from equation (1) is linearly homogeneous in input-prices w and it has $N(N+1)/2 + 2N + 3$ independent parameters, just the right number to be flexible functional form, (Diewert and Wales, 1987).¹⁹ The first set of $N(N+1)/2$ independent terms on the right hand side of equation (1) correspond to the "*Generalised Leontief cost function*" for a constant returns to scale technology with no technological progress, (Diewert).²⁰

The i th input demand function which correspond to equation (1) can be obtained by differentiating C with respect to w_i (using the "*Shephard's lemma*").²¹

The function $w_i^{1/2} w_j^{1/2}$ is concave in w and as a nonnegative sum of concave functions is concave. That function is a nondecreasing in $\square$ follows from the nonnegativity of the parameters $\square_{ij}$. If all $\square_{ij} = 0$ (for i, j) then the above equation reduces to a linear production function.

The production function given by equation exhibits constant returns to scale; we can generalize equation to any degree of returns to scale by:

$$c = f \left(\sum_{i=1}^n \sum_{j=1}^n \gamma_{ij} w_i^{1/2} w_j^{1/2} \right) \quad (1')$$

where, $\square_{ij} = \square_{ji} \geq 0$ and f is a continuous monotonically increasing function which tends to plus infinity and has $f(0) = 0$.

The "*Generalized Leontief cost function*" and the "*Generalized linear production function*" are very useful by providing a second-order approximations to an arbitrary twice differentiable cost function (or

¹⁸Also, we can use the following equation for the *Generalized Leontief Function*:

$$C(w_K, w_L, Y, T) = \alpha_0 + \alpha_Y Y + \alpha_{YY} Y^2 + \sum_{i=1}^n \alpha_i w_i + \sum_{i=1}^n \sum_{j=1}^n \gamma_{ij} w_i^{1/2} w_j^{1/2} + \sum_{i=1}^n \gamma_{it} w_i^{1/2} T + \gamma_t \sum_{i=1}^n w_i^{1/2} T + \gamma_{YY} \sum_{i=1}^n w_i^{1/2} Y^2 + \gamma_{iT} \sum_{i=1}^n w_i^{1/2} T^2 Y + \gamma_{TT} T^2 + \gamma_{YT} T$$

where: where: C is the total cost, Y is the output, w_{ij} the prices of n inputs ($i, j = 1, \dots, n$), and Q_{ij} the n input quantities, T is the time trend.

¹⁹Diewert Erwin and Terence J. Wales (1987) "Flexible Functional Forms and Global Curvature Conditions", *Econometrica*, vol.55:1, January, pp:43-68.

²⁰Diewert Erwin (1971) "An Application of the Shepard Duality Theorem: A Generalized Linear Production Function", *Journal of Political Economy*, vol. 79:3, May/June, pp: 482-507 and also, Diewert Erwin (1974) "Applications of Duality Theory" in Michael D. Intriligator and David A. Kendrick eds., *Frontiers of Quantitative Economics*, vol. II, Amsterdam: North-Holland, pp: 106-171.

²¹ $X_i(w, Y, T) = \sum_{j=1}^n \gamma_{ij} w_i^{1/2} w_j^{1/2} Y + \gamma_i + \gamma_{it} TY + \gamma_t T + \gamma_{YY} Y^2 + \gamma_{TT} T^2 Y$ with ($i = 1, 2, \dots, n$).

The "*Generalised Leontief linear function*" can also be written as: $Y = H(A_{11}K + A_{12}K^{1/2}L^{1/2} + A_{22}L)$, where, A_{ii} 's are parameters and H is a single value increasing function. Also, we assume the *homotheticity hypothesis* for H, (Yasushi Toda).

production function) at a given vector of factor prices or at a given vector of inputs using minimal number of parameters.

As we discussed later, in order to obtain equations that are responsible to estimation, it is convenient to employ the shephard's lemma which states that the optimal cost-minimizing demand for input i can simply be derived by differentiating the cost function with respect to w_i .

Therefore, if we differentiate the equation (1) with respect to w_i yielding the equation (2) and dividing the equation by Y , then yielding the optimal input-output equation (3) denoted by $\square_i$:

$$\alpha_i = \frac{X_i}{Y} = \sum_{j=1}^n \gamma_{ij} (w_j / w_i)^{1/2} \quad (2)$$

$$\frac{\partial C}{\partial w_i} = X_i = Y \left[\sum_{j=1}^n \gamma_{ij} (w_i w_j)^{1/2} \right] \quad (3)$$

when, $i = j$ then $(w_j/w_i)^{1/2}$ is equal to 1 and the $\square_{ij}$ is a constant term in the input-output equation.

Assuming two inputs, such as K = the capital and L = the labour and also Y = the output. The "Generalised Leontief cost-minimising" equations are the followings:

$$\alpha_K = \frac{K}{Y} = \gamma_{KK} + \gamma_{KL} (w_L / w_K)^{1/2} \quad (3)$$

$$\alpha_L = \frac{L}{Y} = \gamma_{LL} + \gamma_{KL} (w_K / w_L)^{1/2} \quad (4)$$

The estimates of all parameters in the "Generalised Leontief cost-function" can be obtained by estimating only the input-output demand equations (3) and (4); this occurs because there is no intercept term in the "Generalised Leontief cost-function" owing to the assumption of the constant returns to scale. Finally, if $\square_{ij}=0$ for all i and j , then the input-output demand equations are independent of the relative input prices and all the cross-price elasticities are equal to zero.

Although equation by equation OLS estimation might appear attractive since the input demand functions (3) and (4) are linear in the parameters, these demand equations have cross-equation symmetry constraints.²² Constant returns to scale restrictions imply the symmetry restrictions (the cost shares sum to one). To implement the "Generalised Leontief model" empirically, a stochastic framework must be specified. An additive disturbance term is appended to each of the input-output equations and is typically assumed that the resulting disturbance vector is independently and identically normally distributed with mean vector zero and constant, nonsingular covariance matrix $\tilde{U}$.

An attractive feature of the "Generalised Leontief cost-function" is that they place no a-priori restrictions on the substitution elasticities. The elasticity of factor substitution measures the responsiveness of the ratio of factor inputs to changes in the ratio of the marginal product of the inputs.

The Hicks-Allen partial elasticities of substitution for a "general dual cost-function" (between inputs i and j in a general functional form with n inputs) can be given as:

$$\square_{ij} = (C * C_{ij}) / (C_i * C_j),$$

where the subscripts i and j refer to the first and the second partial derivatives of the cost function with respect to the input prices w_i and w_j .

In particular, for "Generalised Leontief cost-function" the cross-substitution elasticities are given:

²²Even these constraints hold in the population, for any given sample equation-by-equation OLS estimates will not reveal such restrictions; for example, $\hat{a}_{KL}$ in the K/Y equation estimated by OLS will not necessarily equal $\hat{a}_{LK}$ estimated in the L/Y equation. For a more detailed analysis see Berndt Ernst (1991) The Practice of Econometrics: Class and Contemporary, Addison-Wesley Publishing Company.

$$\sigma_{ij} = \frac{1}{2} - \frac{C \gamma_{ij} (w_i w_j)^{-1/2}}{Y \alpha_i \alpha_j} \quad (5)$$

where $i, j = 1, \dots, n$ (with $i \neq j$).

while the own-substitution elasticities are given as following:

$$\sigma_{ii} = \frac{-\frac{1}{2} C \sum_{\substack{j=1 \\ j \neq i}}^n \gamma_{ij} (w_j^{1/2} w_i^{-3/2})}{Y \alpha_i^2} \quad (6)$$

where $i, j = 1, \dots, n$.

From the other hand in order to estimate the price elasticities with the output quantity and all other input prices fixed²³, we can use the calculation of the following formula:

$$\square_{ij} = S_i * \square_{ij},$$

where S_i is the cost share of the j th input in the total production costs.

For the "Generalised Leontief cost-function" the cross-prices elasticities are computed as following:

$$\varepsilon_{ij} = \frac{1}{2} - \frac{\gamma_{ij} (w_i / w_j)^{-1/2}}{\alpha_i} \quad (7)$$

where $i, j = 1, \dots, n$ (with $i \neq j$).

while the own-prices elasticities are computed as following:

$$\varepsilon_{ii} = \frac{-\frac{1}{2} \sum_{\substack{j=1 \\ j \neq i}}^n \gamma_{ij} (w_i / w_j)^{-1/2}}{\alpha_i} \quad (8)$$

where $i, j = 1, \dots, n$. In order the own-prices elasticity to be negative it is necessary the summation portion of equation (8) to be positive.²⁴

²³ Also, the familiar price elasticities are given by:

$$\varepsilon_{ij} = \frac{\partial \ln X_i}{\partial \ln w_j} = \left(\frac{\partial X_i}{\partial w_j} \right) \left(\frac{w_j}{X_i} \right)$$

see Berbdit Ernst (1991) The Practice of Econometrics: Class and Contemporary, Addison-Wesley Publishing Company.

²⁴ Because from the equations (7) and (8) of elasticity computations are based on the estimated parameters, and the predicted or fitted values of C and $\hat{\alpha}_i, \hat{\alpha}_j$, it is necessary to check the elasticity calculations which always must be hold:

$$\sum_{j=1}^n \varepsilon_{ij} = 0, \text{ (with } i = 1, \dots, n)$$

see Berbdit Ernst (1991) The Practice of Econometrics: Class and Contemporary, Addison-Wesley Publishing Company.

Since the input prices and $\hat{a}_i$ vary between observations, then the estimators of $\hat{\sigma}_{ij}$ and $\hat{a}_{ij}$ will also differ between observations. The price elasticities are not symmetric that means $\hat{a}_{ij} \neq \hat{a}_{ji}$ unlike the Hicks-Allen elasticities and our assumption of $\hat{\sigma}_{ij} = \hat{\sigma}_{ji}$. According to equations (5), (6), (7), and (8) the input i and j are substitutes, independent or complement inputs depending on whether the estimated $\hat{a}_{ij}$ is positive, zero, or negative values. To ensure as its required by theory that the estimated cost function is monotonically increasing and strictly quasi-concave in input prices, one must verify that the fitted values for all the input-output equations are positive and that the $n \times n$ matrix of the $\hat{\sigma}_{ij}$ substitution elasticities is negative semidefinite at each observation. Because the computed elasticities depend on the estimated parameters and therefore are stochastic, the estimated elasticities have also variances and covariances, we should calculate these variances.

3. THE EMPIRICAL RESULTS

The advantages of using a cost function rather than a production approach, in order to estimate the elasticities of substitution may be summarized in the in the followings:(a) the elasticity of substitution is linearly related to the estimated parameters and thus the econometric parameters are more well defined; (b) no matter what the properties of the production function are, the cost function is always linearly homogeneous in the prices and as a result the estimating procedure is more general; (c) the multicollinearity problems which are inherent to the production function approach due to the high correlation between inputs are less severe, because the prices are formed in separate factor markets.²⁵

One of the problems in estimating the elasticity of substitution is that of accurately specifying the production function as well as the type of technical progress. Of course, to impose the cross equation constraints it is necessary to estimate the Zellner's seemingly unrelated estimator (or the so-called ZEF) and we can obtain more efficient estimates if one imposed the cross-equation symmetry constraints in the IZEF equation. We have assumed that the input prices were endogenous and that failure to account lead to a simultaneous bias. One can employ instrumental variable techniques. Equation-by equation two stage least squares (2SLS) estimation would be again inappropriate, since it could not be impose the requisite cross-equation parameter restrictions. Instead of that, one could employ three stage least squares (3SLS) estimation procedure. If one iterated the three stage least squares (3SLS) estimation, the estimated parameters would not be numerically equivalent to those of the full information maximum likelihood technique, even though their asymptotic properties are identical.²⁶

The data used for our estimations come from the annual industrial surveys (AIS) and from the statistical yearbooks (SY) of the National Statistical Service of Greece (NSSG). The data we are use refer to the *large industries* (which correspond to the industries with 20 or more employees persons).²⁷ Output is measured as value added in the large industries, as reported by AIS and SY. The labour is measured as number of employess. The wage rate and salaries correspond to the total labour cost for the large industry. The price of labour was derived by dividing the total labour cost by the number of employess. The prices of capital stock was derived by dividing the value added minus the total labour cost by the capital stock figures. The data on value added and wage rates have been deflated to the constant prices in 1985. These data are available for twenty digit industrial sectors for 32 years (1959-1990). A function of manufacturing sector as a whole estimated using the same data, and each variable is *weighted* by its shares and calculate the averages. We are also using the three stage least square estimators with endogenous lag variables for the Generalised Leontief function.

Table 2.1 Parameter estimations(time-series of Generalised Leontief function): Greece (1959-1990)

	KSH	LSH	$\square_K$	$\square_L$	$\square_{KK}$	$\square_{LL}$	$\square_{KL}$	$\square_{LK}$
20	0.0466	0.0463	0.0421	0.0312	0.0774 (7.67)	-0.0980 (-1.03)	-0.0272 (-4.61)	0.0442 (6.591)
21	0.0459	0.0362	0.0414	0.00509	0.08857	-0.1019	-0.0663	0.1791

²⁵ The cost function approach does not dominate the production function approach; the choice depends on the parameters to be estimated. For example, the reasons much the same as the ones given above the production function approach is preferable when estimates of factor productivities are sought, (Caramanis and Ioannides).

²⁶ Hausman Jerry (1975) "An Instrumental Variable Approach to Full Information Estimates for Linear and Certain Nonlinear Econometric Models", *Journal of Economic Dynamics and Control*, vol.2:1, february, pp:7-46.

²⁷ The following analysis is conducted at the level of "double digit international standard classification of industry" (ISIC). According to this classification of industry we will use the branches (the brackets showing the categories): (20)food, (21)beverages, (22)tobacco, (23)textiles, (24)footwear & wearing apparel, (25)wood & cork,(26)furniture, (27)paper, (28)printing-publishing, (28)leather, (30)rubber and plastic products, (31)chemicals, (32)petroleum, (33)non-metallic mineral products, (34)basic metal industry, (35)metal products, (36)machinery & appliances (37)electrical supplies, (38)transport equipments (39)miscellaneous manufacturing industry.

					(8.711)	(-1.708)	(-5.842)	(9.0111)
22	0.01548	0.0141	0.01516	-0.0427	0.0299 (16.76)	0.00059 (0.044)	-0.0204 (-10.19)	0.0577 (12.62)
23	0.0360	0.0353	0.0330	0.0286	0.0738 (9.448)	-0.0183 (3.234)	-0.0431 (-6.33)	0.0318 (10.55)
24	0.0875	0.1307	0.0805	0.2331	0.1707 (7.384)	-0.0655 (-1.775)	-0.0619 (-4.842)	0.1774 (5.671)
25	0.07221	0.08801	0.0659	0.1101	0.1206 (6.292)	-0.0384 (-0.169)	-0.03431 (-3.658)	0.0820 (4.374)
26	0.06639	0.06835	0.06071	0.05432	0.1197 (9.711)	-0.0212 (-3.382)	-0.2590 (-5.535)	0.1061 (16.28)
27	0.02168	0.03716	0.0334	-0.08531	0.0896 (9.086)	-0.0137 (-2.766)	-0.0715 (-6.469)	0.01907 (7.594)
28	0.03589	0.0333	0.03321	-0.02937	0.0675 (10.46)	-0.01017 (-3.436)	-0.0259 (-6.744)	0.02937 (16.79)
29	0.04018	0.05611	0.03727	0.01360	0.0857 (10.74)	-0.0274 (-2.517)	-0.03646 (-7.096)	0.7362 (8.153)
30	0.07589	0.07356	0.0616	0.1012	0.1053 (4.29)	0.1068 (0.3613)	-0.03395 (-2.375)	0.03638 (1.800)
31	0.0266	0.0309	0.0283	0.0121	0.0442 (6.242)	0.02251 (2.999)	-0.0162 (-3.273)	-0.00576 (0.391)
32	0.0445	0.0254	0.0417	-0.184	0.0778 (8.691)	0.00536 (0.112)	-0.04932 (-5.554)	0.05442 (6.92)
33	0.04176	0.0483	0.0374	0.0259	0.0940 (10.17)	-0.0288 (-3.131)	-0.0592 (-7.376)	0.0426 (9.123)
34	0.0678	0.0648	0.06014	0.0428	0.1402 (5.710)	-0.0241 (-0.882)	-0.0943 (-3.925)	0.0464 (3.497)
35	0.0639	0.0542	0.0490	0.0869	0.0990 (8.165)	-0.0315 (-2.958)	-0.0369 (-5.436)	0.0694 (9.749)
36	0.0426	0.0457	0.0382	0.0453	0.0836 (7.21)	-0.0189 (-2.239)	-0.0189 (-2.23)	0.0447 (8.10)
37	0.0569	0.0432	0.0466	0.0561	0.0912 (5.76)	-0.0572 (-0.491)	-0.0365 (-3.649)	0.0293 (4.12)
38	0.2539	0.0121	0.2195	-0.1018	0.3305 (6.14)	0.0665 (2.10)	-0.2270 (-3.49)	0.00253 (1.796)
39	0.0535	0.0541	0.05151	0.05175	0.0989 (6.79)	-0.0214 (-0.178)	-0.0348 (-4.11)	0.0414 (5.46)
20-39	0.0916	0.0013	0.0832	-0.0066	0.1534 (7.66)	-0.00293 (-1.06)	-0.0542 (-4.61)	0.0129 (6.619)

Note: The numbers in the brackets indicating the t-statistic. KSH and LSH indicate the capital share and labour share, respectively. Note:(*) According to the ISIC classification, we have the branches (the brackets show the categories): (20)food, (21)beverages, (22)tobacco, (23)textiles, (24)footwear and wearing apparel, (25)wood and cork,(26)furniture, (27)paper, (28) printing -publishing, (28)leather, (30)rubber and plastic products, (31)chemicals, (32)petroleum, (33)non-metallic mineral products, (34)basic metal industry, (35) metal products, (36)machinery & appliances (37)electrical supplies, (38)transport equipments (39)miscellaneous industry.

Table 2.2 Substitution and price elasticities of Generalised Leontief function): (1959-1990)

	$\square_{LL}$	$\square_{KK}$	$\square_{KL}$	$\hat{\alpha}_{LL}$	$\hat{\alpha}_{KK}$	$\hat{\alpha}_{LK}$	$\hat{\alpha}_{KL}$	c/l
Foodstuffs(20)	-0.839	-1.101	0.957	-0.44	-0.51	0.44	0.51	c.u
Beverages(21)	-4.222	-0.184	0.875	-0.72	-0.15	0.72	0.15	c.u
Tobacco(22):	-1.699	-0.102	0.412	-0.33	-0.08	0.33	0.08	c.u
Textiles(23):	-1.077	-0.963	1.014	-0.52	-0.49	0.52	0.49	c.s
Footwear & wearing(24):	-0.267	-1.177	0.554	-0.22	-0.28	0.22	0.28	c.u
Wood & cork(25)	-0.299	-1.228	0.596	-0.17	-0.37	0.17	0.37	c.u
Furniture(26):	-0.278	-2.207	0.777	-0.20	-0.57	0.20	0.57	c.u
Paper(27):	-0.347	-0.639	0.459	-0.19	-0.26	0.19	0.26	c.u
Printing-	-0.676	-0.483	0.564	-0.30	-0.25	0.30	0.25	c.u

publishing(28)								
Leather(29):	-0.285	-1.855	0.723	-0.20	-0.51	0.20	0.51	c.u
Rubber & plastics(30):	-0.416	-0.645	0.508	-0.22	-0.28	0.22	0.28	c.u
Chemical(31)	-1.383	-0.427	0.731	-0.46	-0.26	0.46	0.26	c.u
Petroleum(32)	-2.631	-0.403	1.000	-0.17	-0.28	0.17	0.28	c.s
Non-Metallic prds(33):	-0.697	-0.904	0.789	-0.36	-0.42	0.36	0.42	c.u
Basic metal ind.(34):	-0.307	-0.129	0.189	-0.11	-0.07	0.11	0.07	c.u
Metal products (35):	-0.587	-0.738	0.653	-0.30	-0.34	0.30	0.34	c.u
Machinery & appl.(36):	-0.486	-0.952	0.672	-0.28	-0.39	0.28	0.39	c.u
Electrical supplies(37):	-1.070	-0.275	0.529	-0.34	-0.17	0.34	0.17	c.u
Transport equipments(38)	-1.214	-0.339	0.615	-0.40	-0.21	0.40	0.21	c.u
Miscellaneous manuf/ind.(39):	-0.634	-1.265	0.890	-0.36	-0.52	0.36	0.52	c.u
All manufacturing:	-0.285	-0.385	0.309	-0.14	-0.16	0.14	0.16	c.u

Note $\square_{LL}$, $\square_{KK}$, $\square_{KL}$ =indicate the substitution elasticities, $\hat{a}_{LL}$, $\hat{a}_{KK}$, $\hat{a}_{KL}$ =indicate the price elasticities, respectively.

Finally, c/l=indicate the capital-labour saving (where c.u. is the capital-using (or labour saving)); according to David and Van De Klundert, (1965) the technical progress is capital-saving if and only if the elasticity of substitution between capital and labour is less than unity in absolute values.

he parameters estimations $\square_{KK}$, $\square_{KL}$, $\square_{LL}$ can be interpreted as the share elasticities of capital and labour. The negative values of the $\square_{KL}$ from the parameter estimators of Generalised leontief function indicating that the share of capital decreasing in the total cost.

The mean own substitution and price elasticities are negative as it is required. That is, the factor demands are price responsive. Furthermore, in twelve sectors 20, 24, 25, 26, 27, 29, 30, 32, 33, 35, 36 and 39 the share of capital is influenced relatively more after a change in the price of labour. In other words, the demand for capital is less inelastic for the above twelve sectors than the labour demand is. Consequently, these industrial Greek sectors are willing for instance to give up comparatively easier some capital inflows in order to substitute with the relatively cheaper labour inputs.

Table 2.3 Parameter estimations cross-section of Generalised Leontief function in Greece (1959-1990)

	KSH	LSH	$\square_K$	$\square_L$	$\square_{KK}$	$\square_{LL}$	$\square_{KL}$	$\square_{LK}$
1959	0.205	0.256	0.213	1.338	0.978 (0.863)	0.436 (3.73)	0.316 (1.045)	-0.045 (-1.54)
1960	0.185	0.216	0.191	0.982	0.764 (0.770)	0.342 (2.63)	0.327 (1.185)	-0.032 (-0.94)
1961	0.157	0.162	0.160	0.861	0.153 (2.13)	0.230 (4.70)	0.198 (0.10)	-0.159 (-1.57)
1962	0.143	0.146	0.145	0.237	0.152 (2.66)	0.220 (4.73)	-0.186 (-0.12)	-0.199 (-1.95)
1963	0.141	0.128	0.140	-0.308	0.186 (3.374)	0.192 (4.87)	-0.111 (-0.85)	-0.204 (-1.89)
1964	0.123	0.118	0.123	-0.150	0.173 (3.99)	0.185 (4.74)	-0.114 (-1.21)	-0.022 (-1.98)
1965	0.104	0.100	0.104	0.251	0.105 (3.45)	0.190 (6.92)	-0.011 (-0.02)	-0.035 (-3.92)
1966	0.087	0.084	0.087	-0.147	0.100 (2.73)	0.194 (4.067)	-0.026 (-0.35)	-0.044 (-2.47)
1967	0.078	0.077	0.078	-0.055	0.114 (3.54)	0.132 (3.97)	-0.079 (-1.15)	-0.0221 (-1.81)
1968	0.072	0.067	0.073	-0.037	0.108 (4.43)	0.146 (4.31)	-0.075 (-1.50)	-0.030 (-2.47)
1969	0.061	0.055	0.060	0.009	0.098 (4.25)	0.115 (3.31)	-0.078 (-1.68)	-0.0231 (-1.84)

1970	0.049	0.043	0.049	0.031	0.078 (3.19)	0.080 (3.62)	-0.053 (-1.21)	-0.0171 (-1.84)
1971	0.043	0.038	0.042	0.084	0.064 (4.23)	0.096 (4.71)	-0.035 (-1.45)	-0.0320 (-3.00)
1972	0.0397	0.0347	0.0398	0.0832	0.061 (2.79)	0.072 (2.82)	-0.0321 (-0.99)	-0.023 (-1.54)
1973	0.0286	0.0246	0.0284	0.0587	0.041 (3.64)	0.010 (0.373)	-0.018 (-1.16)	0.0756 (0.44)
1974	0.0220	0.0200	0.0219	0.0401	0.0276 (6.00)	0.0264 (2.075)	-0.0678 (-1.26)	-0.0481 (-0.55)
1975	0.0185	0.0198	0.0185	0.0384	0.0260 (6.68)	0.0430 (3.960)	-0.0802 (-1.99)	-0.0188 (-2.19)
1976	0.0146	0.0156	0.0147	0.0196	0.0203 (6.54)	0.0246 (4.69)	-0.0551 (-1.88)	-0.075 (-1.95)
1977	0.0199	0.0133	0.0121	0.0120	0.0229 (10.3)	0.0258 (5.96)	-0.010 (-5.09)	-0.011 (-3.15)
1978	0.0090	0.0087	0.0091	0.0102	0.0139 (5.37)	0.0186 (4.43)	-0.046 (-1.85)	-0.0855 (-2.43)
1979	0.0067	0.0063	0.0068	0.0122	0.0971 (6.50)	0.0166 (6.13)	-0.0262 (-2.01)	-0.0103 (-3.94)
1980	0.0053	0.0046	0.0053	0.0046	0.053 (12.8)	0.046 (10.0)	0.0001 (1.00)	0.0001 (1.01)
1981	0.0053	0.0060	0.0052	0.0158	0.0692 (5.24)	0.0200 (7.14)	-0.0103 (-1.27)	-0.0205 (-5.16)
1982	0.0037	0.0064	0.0035	0.0062	0.050 (10.1)	0.041 (10.0)	0.0001 (1.00)	0.0001 (1.01)
1983	0.0035	0.0047	0.0035	0.0105	0.0444 (5.14)	0.0131 (3.771)	-0.004 (-1.03)	-0.0157 (-2.48)
1984	0.0028	0.0040	0.0028	0.0081	0.0347 (3.97)	0.0712 (3.58)	-0.0034 (-0.77)	-0.052 (-1.61)
1985	0.0024	0.0032	0.0024	0.0061	0.0217 (6.88)	0.0352 (1.32)	0.1031 (0.84)	-0.003 (-0.07)
1986	0.0021	0.0025	0.0021	0.0058	0.0186 (2.49)	0.0294 (1.11)	0.0015 (0.452)	-0.007 (-0.15)
1987	0.0018	0.0021	0.0019	0.0048	0.0208 (3.91)	0.0295 (1.75)	-0.0007 (-0.34)	-0.0166 (-0.49)
1988	0.0015	0.0017	0.0015	0.0039	0.0158 (3.91)	0.0237 (1.83)	-0.0001 (-0.09)	-0.015 (-0.56)
1989	0.0013	0.0014	0.0013	0.0031	0.0142 (3.83)	0.0160 (1.32)	-0.0003 (-0.23)	-0.0044 (-0.16)
1990	0.0010	0.0013	0.0010	0.0026	0.0142 (4.30)	0.0142 (1.49)	-0.0009 (-1.01)	-0.0037 (-0.14)

Note: The numbers in the brackets indicating the t-statistic. This analysis indicate all industries by each year, (the variables are weighted shares). KSH and LSH indicate the capital share and labour share, respectively

In order to implement the "Generalised Leontief model" empirically, a stochastic framework must be specified.²⁸ We are using the "Shephard duality theorem"; according to which if we estimate econometrically the parameters of a cost function then this is equivalent to estimate the parameters which describe the underlying production possibility sets (assuming that the producers are behaving competitively in the factor markets).²⁹

²⁸ A disturbance term must specified in each of the input-output equations (4) and (5) and also is assumed that the disturbance vector is independently and identical normally distributed with mean vector zero and constant nonsingular covariance matrix W. These disturbancy terms could simply reflect optimization errors on the part of industries, (Berndt, 1991). Since the input-demand functions are linear in the parameters and these demand equations have cross-equation symmetry constrains, then the OLS estimation equation by equation appear more attractive. However, we can use the "Zellner's seemingly unrelated estimator" (ZEF which called and "seemingly unrelated regression estimator (SUR) or the minimum ch-square estimator"). The different parameter estimates than those form equation by equation OLS result from the following reasons:(a)the disturbances across the input-output equations are contemporaneously correlated which imply that the disturbance covariance matrix is nondiagonal.(b)each of the equations (4) and (5) contains different regressors. For these reasons the ZEF estimator will provide more efficiently estimates of parameters rather than the OLS.The "Zellner's seemingly unrelated estimator" (ZEF) uses equation by equation OLS to obtain an estimate of the disturbance covariance matrix and then does generalized the least squares, (Berndt, 1991).

²⁹The duality property between cost and production function was first introduced by Shephard (1953). Given that a cost function satisfying the certain regulatory conditions, we can use a cost function to define a production function which in turn may be used to derive our original cost function, (Diewert).

We employ data for the twenty-digit industrial sectors in Greece. Table 2.1 and Table 2.2 are showing the estimation results of Generalised Leontief function. In general, the estimates are statistically significant.

These parameters describing the implications of patterns of substitution for the relative distribution of output between capital and labour inputs. A positive share elasticity implies that the corresponding value share increases with an increase in quantity, while a negative share elasticity implies that the value share decreases with an increase in the quantity.

The parameters σ_K and σ_L can be interpreted as average value shares of capital and labour inputs; these parameters are positive and also statistically significant (except in the case of σ_L for the sectors of tobacco (sector 22), paper (sector 27), printing-publishing (sector 28), petroleum (sector 32), transport equipments (sector 38) and total industry equipments (sector 20-39) which are negative.

Table 2.4 Substitution and price elasticities, technical change and scale for period 1959-1990

	σ_{LL}	σ_{KK}	σ_{KL}	$\hat{\alpha}_{LL}$	$\hat{\alpha}_{KK}$	$\hat{\alpha}_{LK}$	$\hat{\alpha}_{KL}$	c/l
1959	-0.285	-0.385	0.309	-0.142	-0.166	0.142	0.166	c.s
1960	-0.409	-0.411	0.368	-0.185	-0.187	0.181	0.187	c.s
1961	14.605	-0.150	0.553	-0.488	-0.064	0.488	0.064	c.s
1962	5.8562	-0.013	0.277	-0.279	0.0027	0.279	-0.0027	c.s
1963	-0.226	-0.212	0.215	-0.111	-0.103	0.111	0.1038	c.s
1964	-0.371	-0.286	0.294	-0.157	-0.137	0.157	0.137	c.s
1965	-0.461	-0.379	0.375	-0.196	-0.178	0.196	0.178	c.s
1966	-0.276	-0.273	0.256	-0.128	-0.127	0.128	0.127	c.s
1967	-0.265	-0.220	0.224	-5.039	-4.194	4.270	-0.117	c.s
1968	-0.190	-0.189	0.183	-0.092	-0.090	0.092	0.0902	c.s
1969	-0.245	-0.247	0.242	-0.123	-0.118	0.089	0.0882	c.s
1970	-0.524	-0.355	0.398	-0.212	-0.180	0.218	0.180	c.s
1971	0.0462	-0.082	0.058	-0.019	-0.038	0.019	0.038	c.s
1972	0.231	0.0714	-0.08	0.058	0.0311	-0.05	-0.03	c.s
1973	-0.38	-0.224	0.260	-0.14	-0.113	0.146	0.113	c.s
1974	-0.45	-0.276	0.308	-0.17	-0.135	0.173	0.135	c.s
1975	-0.61	-0.466	0.455	-0.24	-0.210	0.245	0.210	c.s
1976	-0.39	-0.319	0.321	-0.16	-0.153	0.168	0.153	c.s
1977	-0.02	-0.081	0.067	-0.02	-0.038	0.028	0.038	c.s
1978	-0.03	-0.136	0.124	-0.05	-0.067	0.057	0.067	c.s
1979	-0.29	-0.20	0.229	-0.12	-0.103	0.125	0.103	c.s
1981	-0.45	-0.39	0.385	-0.20	-0.183	0.201	0.183	c.s
1982	-0.51	-0.80	0.559	-0.25	-0.305	0.254	0.305	c.s
1983	-0.69	-0.80	0.639	-0.31	-0.324	0.314	0.324	c.s
1984	-0.53	-0.54	0.488	-0.24	-0.248	0.240	0.248	c.s
1985	-1.08	-1.69	1.054	-0.49	-0.558	0.496	0.558	c.s
1986	-0.41	-0.28	0.331	-0.17	-0.153	0.178	0.153	c.s
1987	-0.35	-0.25	0.283	-0.15	-0.132	0.150	0.132	c.s
1988	-0.47	-0.35	0.011	-0.19	-0.167	0.193	0.167	c.s
1989	-0.63	-0.48	0.488	-0.26	-0.227	0.260	0.227	c.s
1990	-0.84	-0.84	0.739	-0.37	-0.366	0.372	0.366	c.s

Note: σ_{LL} , σ_{KK} , σ_{KL} =indicate the substitution elasticities, $\hat{\alpha}_{LL}$, $\hat{\alpha}_{KK}$, $\hat{\alpha}_{KL}$ =indicate the price elasticities, respectively. Finally, c/l=indicate the capital-labour saving where c.u. is the capital-using (or labour saving)); according to David and Van De Klundert, (1965) the technical progress is capital-saving if and only if the elasticity of substitution between capital and labour is less than unity in absolute values.

Furthermore, Tables 2.3 and 2.4 illustrates the annually cross-section estimation of Generalised Leontief Cost Function for Greek industry. To interpret the estimates of these parameters it is useful to recall that if the production function is increasing in capital and labour inputs then the average value shares are non negative. These conditions are satisfied by our estimations. The parameters σ_{KK} , σ_{KL} , σ_{LL} can be interpreted as the constant share elasticities. A share elasticity equal to zero implies that the corresponding value share is independent of quantity; we are assuming that σ_{KK} , σ_{LL} are zero. The parameter σ_{KL} , found that is negative (as in the previous estimation of the Generalised Leontief function) but however, in the most of the sectors these results are not statistically significant. In addition, from the translog function we can get some estimations for the substitution elasticities (σ_{LL} , σ_{KK} , σ_{KL}). The σ_{KK} , σ_{KL} have to be negative because of the

"demand-law" for the inputs. If the σ_{KL} which is the substitution elasticity between K and L is positive, then the K and L are compliments, or otherwise they are substitutes. If the σ_{KL} is less than unity, then that means the "technical change is capital-saving" (or otherwise "labour-saving").

Summarizing, we can say that the application of the translog-function is giving us more opportunities to get better and more precisely estimations for the substitution elasticities, the rate of technological change and the different parameters.

Lianos Kintis used a constant elasticity of substitution (CES) production function was used to estimate the elasticity of substitution in the manufacturing sector of the Greek economy. Lianos used two alternative formulations of the production functions derived estimates of the elasticity of substitution between capital and labour in Greek manufacturing.³⁰

These studies of Lianos and Kintis found that the technological progress in Greece was biased. Another study using a technique proposed by Sato which postulates the existence of a production function of the factor augmenting type relating output to capital and labour measuring in efficiency units and in his study concludes that in the Greek manufacturing sector the technological progress is capital-using. In another model E. Panas (1986) following in his study the "Bergstrom-Melander-Kalt Model", presenting some further evidence on the degree of substitution between capital and labour and the estimates of capital-labour augmentation coefficients, (using an autoregressive SURE Model).

The results and estimates of Panas' SURE model is different from those of Kintis: (on the basis of SURE estimates one would reject the capital-using hypothesis in favor of labour using hypothesis in two industries 25 and 27 and this reveal that technological progress in a number of industries is completely different from that reported by Kintis). Panas was found that the bias of technological progress in Greek manufacturing is not neutral: (in fact, during the period 1958-1975 the increase in the wage rate is greater than that of the profit rate and then we can say that the entrepreneur will be included to save labour).

Finally, one can deal with alternatives flexible functional forms such as the translog cost function³¹ or even more general forms such as the meta-production function.³²

4. CONCLUSIONS

The form of technological progress has been one of the main objectives of economic policy. When a firm can achieve the same output with different combinations of factors of production, it may substitute one factor for one or more others. The substitutability in a industry is depending from the availability of capital, labour factors, from the level of adoption and diffusion of new technologies, from the level of know-how etc. The substitution of capital for labour corresponding to an increase in the importance level of capital compared with the labour in the course of production process (for instance, the use of new techniques, methods etc.). The appropriateness of technology can be measured through the substitutability of labour-capital variables.

The substitution effect is linked to the characteristics of production technology. It increases with the possibility of substitution between factors of production. Such possibilities are measured by their elasticity of substitution. A concept which most directly reflects the technical constraints inherent in production processes.

In this paper we tried to analyse a flexible functional form for Greek industry and to examine the substitutability between capital, and labour factors. Also, we should try to test the complementary/substitution effects, to measure if the constant return of scales hold for all industry. We can examine the relationship between inputs in industrial production, (as inputs we can assumed the capital, and labour).

The main obstacle facing empirical studies of R&D is the lack of information about the rate of R&D depreciation especially in two-digit industries. Most of the studies used an R&D depreciation rate 10% or even

³⁰The first one corporate Hicks-neutral technical change:

$$Y=A(t)F(K,L) \quad (1),$$

in the equation (1) the term A(t) is the term of technical progress. The second allows for factor augmenting technical progress:

$$Y=F(A(t)K,B(t)L), \quad (2),$$

in the equation (2) A(t) and B(t) are the capital and labour augmenting factors. Also, the output is measured as a value-added, Labour is measured in manhours of works supplied by workers, employees and unpaid persons and the wage (W) was obtained by dividing the wage bill by the number of man-hours of paid workers and employees.

³¹ Korres G. and Paraskevopoulos Y.: (1996) "The effects of technical change and foreign direct investment on the evaluation of Greek manufacturing", paper presented at the conference of the *European Firm in the Global Economy* at the Economic Society of Thessaloniki, Aristotelion University (in Greece).

³² Korres G: "Technical change and Productivity Growth: an empirical evidence from European countries", A Book published by Avebury-Ashgate publishing company in London (1996).

we can take an alternative R&D depreciation rates of 5 % and 20 %. We can review some of the results which have been obtained from the regression for Greek economy for the period from 1973 to 1984. Unfortunately, was available only aggregate data for Greek economy (especially concerning the variable x_3 of R&TD).

The results are showing that the capital variable is complementary with those of labour and R&TD, while the labour and R&TD variables are found to be substitutes. Positive signs imply a direct relationship, while negative signs reflect a combination of the elasticity of input demand and strong complementary with other factors. Finally, the different statistical tests shows that variables are non-CRST, (a symmetry/constant returns to scale holds).

Greece was one of those countries which having a rapid-growth in the period after the second world war when the transformation in the structure of economy from agricultural to the new-industrialized country took place. The period from 1950 until the end of 1960s has been characterized by a rapid expansion of growth rates. In this period there was a new *technological orientation* of more sophisticated products, while most of the technological inputs are transferred from abroad. The period after 1974, show a decline in economic growth. This is more obvious for the period between 1974 and 1980. The decade of the 1980s can be characterized as the start of a *new era* for technological activities, especially after the accession of Greece to the European Union.

The results indicate that most of the industrial sectors are capital using intensive (or labour saving) which can be interpreted in accordance to the previous analysis that the technological inputs (such as the imported capital goods and the transferred technologies) were not appropriate to the local necessities and did not fit the availability of market resources.

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