

# A MULTIVARIATE LOGISTIC MODEL FOR COMPETING TECHNOLOGICAL PRODUCTS IN A MARKET

**Ioannis Dimotikalis**

Technological Education Institute of CRETE, Dept. of Finance and Insurance  
Aghios Nikolaos, CRETE  
[jdimm@finance.teicrete.gr](mailto:jdimm@finance.teicrete.gr)

## Abstract

*We propose a multivariate extension of discrete logistic model to represent the whole competition of  $m$  technological products, coming from a technology, in a specific market. The relation to Lotka-Volterra model it is examined and used for an improvement of the model. We apply the proposed model to the data set of the Greek mobile phones market data, consisting of 3 competitors, to evaluate model performance in fitting and forecasting. In both markets the model fitting and forecasting performance is acceptable and encouraging. Finally some remarks, conclusions and future tasks are discussed both methodological and practical about multivariate logistic model.*

## 1. Introduction

The progress of technology results to the design and introduction of new technological products that finally appear to every day life of citizens in society. In the case of technological products introduced to the market for the customer, usually more than one companies or organizations produces and supply the product. For the whole market of the technology an innovation diffusion process is expected until the product or technology adopted by all potential customers. A competition for maximum market share and profit between those competing products and companies take place. Improvements of the products lead to substitution of earlier products by superior products. An example is the analysis of computers market evolution during last decades [1, 2].

To study the adoption process of the technological products one can use the known technological forecasting models such Logistic, Bass, Fisher-Pry, etc. Lately are presented and are available even stochastic versions of those models [3]. A problem arise when the products of technology sold by more than one company, then the competing products on the same market can studied by summing up all the products data or product per product individually. It is also known from literature specific models of technological substitution and multiple generation products [2, 4], but this is not the main case studied here.

It is useful from many points of views to study the whole competition of the competing technological products and companies. From the companies point of view it is important to take into account the competition and competitors, from the technology point of view it is important to examine if the competition change the adoption process and how this done. Finally from the customer point of view it is generally accepted that competition is offering better products, prices or both. For example, in the report of OECD [5] about mobile telecommunications worldwide, it is pointed out that the penetration rate of mobile phones is higher in countries with more competitors in the market.

In this paper we use the discrete form of logistic model to represent mathematically the whole competition of  $m$  technological products of a technology in a particular market. Some basic considerations are the potential of the market, the potential adopters of technology (buyers of the products), known as saturation point in technological forecasting. The products are buoyed by imitation of non-customers to customers (adopters). A customer buys the product that seem to him more attractive or suitable to particular personal needs.

Because of multiple products or companies who offer the product, the whole market data is a multivariate time series. To study the whole competition at once, a multivariate model is needed to represent the evolution of all products in the market. Each product (or company) represented by a variable of the multivariate model. The common point of these products is that shares the same market and serves a particular need of the customers. Modis [6, 7] proposed the use of two variables Lotka-Volterra model to study such data.

In section 2 starting from the logistic model in discrete form we introduce a multivariate formulation of the model for  $m$  competing products in the same market by some suggestions, transformations and algebraic calculations. A fitting method is proposed to simplify model application to real data sets, using spreadsheet functions and tools. At section 3 the model applied to the data set of Greek mobile phones market subscribers (cumulative quarterly data), which consist of three competitors. As well it is examined the performance of the proposed model in fitting and forecasting. Finally in the last section we summarize the conclusions for

the proposed model. Also discussed theoretical and practical remarks concerning the application of the model to real data. Future tasks and objectives close this work.

## 2. The Discrete Multivariate Logistic Model

A classic formulation of normalized univariate (single variable) discrete logistic model [9, 10, 11] is:

$$x_{n+1} = x_n b(1 - x_n) \quad (1)$$

where  $x$  variable take values in the interval  $(0,1)$ . For the case of a technological product, the variable  $x_n$  measures for example the buyers of the product (as market share percentage),  $b$  is the control parameter of growth (imitation coefficient) and the term  $(1-x_n)$  measures the remaining potential customers (buyers) on the market. The evolution of the product in the market depends on all previous terms and parameters. For logistic equation (1) it is known that the model values  $x_n$  converges to the value  $(1-1/b)$  as  $n$  goes to infinity (if  $1 < b < 3$ ), independent from the starting value  $x_0$  [10,11]. Because sometimes is difficult to transform meaningful the available data to vary between 0 and 1, a different formulation of logistic model is useful. An appropriate transformation of the model is the following such that  $x_n$  can get values between 0 and  $K$  (positive value):

$$x_{n+1} = x_n b \left( \frac{K - x_n}{K} \right) \quad (2)$$

after some calculation yields:

$$x_{n+1} = bx_n - \frac{b}{K} x_n^2 \quad (3)$$

In this form the parameter  $K$  represent the carrying capacity (saturation level) of the market, all the potential consumers of the product. The term  $\frac{K - x_n}{K}$  of eq. (2) is the substitution of the term  $(1-x_n)$  in equation (1) and represents the remaining potential of the market. The values of  $x$  now converge to the value  $K(1-1/b)$  as  $n$  grows.

This formulation of the model remains problematic because the model cannot take all the values in the whole interval  $(0, K)$  but only in the interval  $(0, K \frac{b-1}{b})$ . It is preferable to have the model written in a form such that  $x$  can take values in the whole interval  $(0, K)$ , and saturates to market size  $K$ .

Observing that the limit value of  $x$  in eq. 2 is  $K \frac{b-1}{b}$ , we introduce a new transformation of logistic model.

By using a new parameter  $M$  such that  $M = K \frac{b-1}{b}$  in equation (2), replacing the  $K$  parameter  $K = \frac{Mb}{b-1}$  yields the logistic model of the form:

$$x_{n+1} = x_n \frac{(b-1)}{M} \left[ \frac{Mb}{(b-1)} - x_n \right] \quad (4)$$

after some simplifications:

$$x_{n+1} = bx_n - \frac{(b-1)}{M} x_n^2 \quad (5)$$

Now the limit value of  $x$  variable after the required iterations is  $M$ , the real saturation limit (market size) of the product diffusion to the market.

### 2.1 Building the Multivariate Logistic Model

In equation (4) of the logistic model the term  $\left[ \frac{Mb}{(b-1)} - x_n \right]$  represents the remaining potential of the market. If there is another competing product  $y$  in the same market, then in the previous term of equation (4)

is needed to introduce the second competitor (e.g. the y competitor value in the market). A simple way to do that is to use the term  $\left[ \frac{Mb}{(b-1)} - x_n - y_n \right]$ , inserting the  $y_n$  value of product y. By this term we take in consideration the true remaining potential of the market for product x and y both. The evolution in the market of the y product can be handled by a similar equation of (4) and (5). The system of differences equations for the two competing products x, y in the market yields:

$$\begin{aligned} x_{n+1} &= x_n \frac{(b_x - 1)}{M} \left[ \frac{Mb_x}{(b_x - 1)} - x_n - y_n \right] \\ y_{n+1} &= y_n \frac{(b_y - 1)}{M} \left[ \frac{Mb_y}{(b_y - 1)} - x_n - y_n \right] \end{aligned} \quad (6)$$

after some calculations resulting the system of differences equations below:

$$\begin{aligned} x_{n+1} &= b_x x_n - \frac{(b_x - 1)}{M} (x_n^2 + y_n x_n) \\ y_{n+1} &= b_y y_n - \frac{(b_y - 1)}{M} (x_n y_n + y_n^2) \end{aligned} \quad (7)$$

is reasonable to suggest that the values of parameter b for each product x,y generally are different,  $b_x$ ,  $b_y$  respectively. By this system of differences equations we can represent and examine the competition of two products in the same market. The common parameter in the system of equations (7) is the parameter M, the size of the market, because competitors compete each other for this. We call eq. 7 the Multivariate Logistic (ML) model with 2 products (variables).

## 2.2 Generalization to Three and More Competitors in the Market

Using the same approach that above to pass from univariate logistic model to competition of 2 products in a market (from eq. 4 to eq. 6), it is easy to arrive at 3 products x, y, z Multivariate Logistic model. Simply incorporate the third product z value  $z_n$  in equation (6).

The system of difference equations for the case of three products, competing in a market, in discrete form is:

$$\begin{aligned} x_{n+1} &= x_n \frac{(b_x - 1)}{M} \left[ \frac{Mb_x}{(b_x - 1)} - x_n - y_n - z_n \right] \\ y_{n+1} &= y_n \frac{(b_y - 1)}{M} \left[ \frac{Mb_y}{(b_y - 1)} - x_n - y_n - z_n \right] \\ z_{n+1} &= z_n \frac{(b_z - 1)}{M} \left[ \frac{Mb_z}{(b_z - 1)} - x_n - y_n - z_n \right] \end{aligned} \quad (8)$$

the Multivariate Logistic model with 3 products in simpler form becomes:

$$\begin{aligned} x_{n+1} &= b_x x_n - \frac{(b_x - 1)}{M} (x_n^2 + y_n x_n + z_n x_n) \\ y_{n+1} &= b_y y_n - \frac{(b_y - 1)}{M} (x_n y_n + y_n^2 + z_n y_n) \\ z_{n+1} &= b_z z_n - \frac{(b_z - 1)}{M} (x_n z_n + y_n z_n + z_n^2) \end{aligned} \quad (9)$$

The associated Extended Multivariate Logistic model derived simply introducing 3 more parameters  $a_x$ ,  $a_y$ ,  $a_z$  in system of equation (9):

$$\begin{aligned}x_{n+1} &= a_x x_n - \frac{(b_x - 1)}{M} (x_n^2 + y_n x_n + z_n x_n) \\y_{n+1} &= a_y y_n - \frac{(b_y - 1)}{M} (x_n y_n + y_n^2 + z_n y_n) \\z_{n+1} &= a_z z_n - \frac{(b_z - 1)}{M} (x_n z_n + y_n z_n + z_n^2)\end{aligned}\quad (10)$$

This approach can be extended to  $m$  different products  $x_1, x_2, x_3, \dots, x_m$  of a particular technology in a market using the equation:

$$x_{i,n+1} = x_{i,n} \frac{(b_x - 1)}{M} \left[ \frac{M b_x}{(b_x - 1)} - \sum_{j=1}^m x_{j,n} \right] \quad i,j=1,2,\dots,m \text{ products} \quad (11)$$

in simplified form the system becomes:

$$x_{i,n+1} = b_i x_{i,n} - \frac{(b_i - 1)}{M} \sum_{j=1}^m x_{i,n} x_{j,n} \quad i,j=1,2,\dots,m \text{ products} \quad (12)$$

By this simplified approach of the Multivariate Logistic model, the growth of a product  $x_i$  depends on current product position  $x_{i,n}$  in the market, the parameter of product growth rate  $b_i$  (imitation coefficient or competitiveness coefficient) and the remaining market potential (the total market size  $M$  minus the sum of market positions of all competing products).

The control parameter  $b_i$  is the important parameter for the evolution of each product to the market. In cases of  $m$  products competition it is reasonable to suppose that the final market position of each product  $x_i$  expected to be analogous of the value of  $b_i$  (and secondly to starting value  $x_{i,0}$ ). Because at every time period each product  $x_i$  gains from the remaining potential of the market a fraction analogous to  $b_i$ , the parameter  $b$  also measures the competitiveness of each product.

The Extended Multivariate Logistic (EML) model for  $m$  competitors in a market has an analogous equation of (12):

$$x_{i,n+1} = a_i x_{i,n} - \frac{(b_i - 1)}{M} \sum_{j=1}^m x_{i,n} x_{j,n} \quad i,j=1,2,\dots,m \text{ products} \quad (13)$$

This version of the multivariate logistic model has totally  $2m+1$  parameters (and  $m$  "hidden" parameters for the starting values). Parameters  $a_i$  holds the growth of each product and parameters  $b_i$  the market limits and competition.

### 2.3 Fitting the Multivariate Logistic Model to Data Sets

A model is useful if it is easy to apply to real data sets. Here we present a simple method to apply the proposed ML and EML model to a market data set. To fit a model to real data the simplest tool is to use the available software utilities and methods. Data today usually exist in spreadsheet worksheets (or easily transferred to), an easy way to fit the proposed model is to use the solver tool of a spreadsheet. The solver tool or command can find the appropriate values of parameters even in nonlinear relations (model).

To use the solver tool of a spreadsheet needed to write the formula for the model estimations next to real data values column of each product (competitor), with absolute reference to spreadsheet cells containing the values of the model parameters. Then the Squared Errors of model estimations calculated and the Sum of the Squared Errors (SSE) of each product curve. Using the solver tool we ask to minimize the total SSE (sum of all products curves errors), by changing the values of model parameters  $a_i, b_i, M$  and initial values  $x_0$  (actually the values in associated spreadsheet cells).

Because of nonlinearity of the ML and EML model it is necessary to have good starting values of the parameters. In the spreadsheet worksheet it is easy to find good starting values by experimentation, because entering starting values to parameters cells the statistics of fit calculated immediately. In practice required to tested different starting values of parameters, to ensure that solver algorithm converges to best set of parameter values (solution) for the data and not to a local minimum [15].

A disadvantage and theoretical problem of the proposed fitting method is the absence of model parameters and fitting statistics (like Standard Error). But user can estimate the error of fitting and related statistics such

that SSE, Mean Squared Error (MSE), Mean Absolute Percentage Error (MAPE) and Mean Absolute Deviation (MAD). From a practical point of view MAPE and MAD is easier to understand by practitioners and decision-makers and will be used to next section.

### 3. Application of the Model to Greek Mobile Phones Markets

Mobile telecommunications is a new technology and hold much attention from scientists of many fields. The diffusion of mobile phones has already studied extensively [13, 5]. In the case of data used here the product actually defined as the subscription of the customer to a mobile phones telecommunications company. Because this technology needs a network to operate, the governments worldwide apply licenses to a limited number of companies that offer mobile telecommunications to customers. Usually market starts to operate with two competitors and new entrances performed later by auctions of the new license. In most European countries today operates 3, 4, or 5 competitors in their markets [5].

#### 3.1 Application to Greek Market Data Set of Mobile Phones Subscribers

The Greek market of mobile phones starts to operate in July 1993 with two competitors, Panafon and Telestet, both with the GSM 900 MHZ system of cellular telephony. The market data where collected from press releases and financial statements of the companies, that exist on their Internet sites. All the data used in this study are available from the author upon request. The data are quarterly cumulative subscribers (in thousands) for the time period Dec 1995- Dec 2000 of each company at the end of each quarter. Data before Dec 1997 are available on yearly basis only. Third competitor Cosmote, a subsidiary of public Greek Telecommunications Organization (OTE), starts to operate later in May 1998.

To apply the Multivariate Logistic model we use the equation (9) for three competing products (companies). The data for each firm start at different time period, for ours formulation of the ML model this is not a problem, in figure 3 is presented the fitting of the model to all available data of Greek mobile phones market. The fit to the Panafon and Telestet data starts at Dec 1995, to Cosmote data at June 1998. As metioned above Cosmote start operate at May 1998, at June reported only about 74 thousands of subscribers. The company already had pre-subscriptions of about 200 thousands of persons, but in June 1998 probably for practical reasons the company did not succed to fill all those pre-orders. The vertical dashed line in figure 3 separate the fitting and forecasting period of the model curves and data. The fit is good and the model predict that in next year Cosmote overpass Panafon subscribers and becomes market leader.

The predicted final market size is  $M=7508.2$  thousands of subscribers and will be reached after about two years (the total population of Greece is about 10500 thousands). The detailed data for parameters values of the ML model fit to Dec 1995-Dec 2000 data, presented in the third column of table 1 under the label ML model. More precicelly in table 1 presented the total fit statistics, the predicted total market size  $M$  (third line) and the total SSE (forth line) of all competitors data fit. To next lines of the table presented the parameters values of model and fitting statistics for each competitor data.

An interesting point is in the values of control parameter  $b$ , Cosmote has the bigger value  $b=1.517$  and that's why predicted to dominate the market next year (finally predicted to reach 3042.3 thousands of subscribers). Follows Telestet with  $b=1.217$ , but his position predicted as third (1931.1 thousands of subscribers).

To next forth column of table 1 are presented the results of EML model fitting to data. The fitting performance is better than ML model (in terms of SSE, MAD and MAPE), but mainly produced by the significantly better fitting to Telestet data, as can be shown in table 1. For eample the  $SSE=4251$  of EML model to Telestet data is about half of  $SSE=8524$  of ML model to same data. In fifth column of table 1 we present the results of applications of univariate logistic model to each competitor data separetely.

From univariate logistic model application market leader predicted Panafon with value of  $M=2630$  slighly bigger than 2546 of Cosmote. But from the multivariate model the final state predicted totally different, final market size  $M=3042$  for Cosmote and 2535 for Panafon.

But certainly this is against the origin of competition, competitors is natural to react to this situation. In fact in Greek mobile phones market at Jun 2000 when Cosmote pass Telestet subscribers (see figure 1), Telestet reduce prices by 20%, Panafon follows to lower prices and Cosmote react setting there prices equal to competitors.

The last entrant in Greek market, Cosmote, lunch the market in May 1998. To examine the full competition period in last three columns of table 1 presented the results of applications of the 3 models (ML, EML, univariate) to this part of data (Jun 1998-Dec 2000). The fit and statistics of ML model is very similar to that of ML for all the data in third column of table 1. The values of  $b$  and  $M$  parameters remain about the same,

statistics of fit also (except the SSE value, but now there are fewer data). Most significant improvement observed to Telestet data where MAD fall to 11.24% from 19.82 and MAPE to 1.09% from 5.25%.

In EML model (seventh column of table 1) observed significant differences from that of all data EML. First is that predict final Telestet dominance after a long period of Panafon dominance. The fit and predictions (projections) presented in figure 2, where forecasting period is extended to shown this complicated situation. A natural question concerns the comparison with the results obtained by application of the univariate logistic model to the total market data (sum of all competitors' subscribers). Applying the proposed univariate logistic model (eq. 3) to the sum of market data we obtain  $M=8724$ ,  $b=1.244$ ,  $SSE=56503$ ,  $MAD=50.55$ ,  $MAPE=5.45\%$ .

It is interesting to examine the forecasting ability and performance of the proposed model. A part of our available data are used to evaluate the success of predictions by the models, to the rest unused part of data in fitting. In Table 2 presented the predictions and their statistics of the models (ML, EML, univariate) to all except the last four data values (year 2000 data). For each competitor the four first lines present the predicted values by the models, the next two lines  $MAPE_{1-2}$  statistic for the first 2 predictions (Mar 2000 and Jun 2000) and  $MAPE_{1-4}$  for the whole 4 predictions of year 2000. Also presented the  $MAD_{1-2}$  for the first 2 predictions and  $MAD_{1-4}$  for the all year 2000 quarterly predictions. Those price changes expected to affect the market and model forecasting performance. For the whole market, all models had similar predictions and statistics, but for the whole year the better was ML model applied to all data Dec 1995-Dec1999 followed by ML model applied on data of Jun 1998-Dec 1999.

#### **4. Conclusions and future tasks**

We have presented a multivariate adaptive logistic model capable to cope with the competition of technological products in a market during introduction. The proposed model can apply to any number of competitors in a market. The total number of model parameters for EML model is  $2m+1$ , where  $m$  is the number of competing products. We also propose a method to fit the model using a solver tool available to anyone with access to a computer and a spreadsheet application.

The ML and EML models are applied successfully to Greek mobile phones market cumulative subscribers. The evaluation of models forecasting ability performed for those data is encouraging and illustrates the characteristics and advantages of the proposed model. A certain advantage of multivariate logistic model is the ability to set the market size parameter  $M$  easier than univariate logistic model, in the presence of competition. The main remaining problem is changes in the competition status (for example price changes) of the market. Those changes cannot affect immediately the model parameters values and can lead to problematic predictions.

A theoretical improvement of the EML model is to investigate and use the relation with the Bass model in discrete form [11]. It looks interesting to build and test a multivariate Bass model.

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### Appendix

**Table 1:** Model parameters and fitting statistics values of Multivariate Logistic (ML), Extended ML and univariate Logistic fit, to Greek mobile phones market cumulative subscribers by company

|            | Parameters and Statistics | Dec 1995-Dec 2000 DATA |                               |                     | Jun 1998-Dec 2000 DATA |                               |                     |
|------------|---------------------------|------------------------|-------------------------------|---------------------|------------------------|-------------------------------|---------------------|
|            |                           | ML model               | EML model                     | Univariate Logistic | ML model               | EML model                     | Univariate Logistic |
| All Market | M                         | 7508.2                 | 7743.7                        | 7503.5              | 7487.7                 | 8410                          | 7732.0              |
|            | SSE                       | 39609                  | 38219                         | 38623               | 27767                  | 25457                         | 27220               |
| PANAFON    | M                         | 2534.8                 | 2493.3<br>( $\rightarrow 0$ ) | 2630.4              | 2505.4                 | 3609.8<br>( $\rightarrow 0$ ) | 2920.0              |
|            | b                         | 1.189                  | 1.202                         | 1.238               | 1.184                  | 1.178                         | 1.205               |
|            | a                         | -                      | 1.192                         | -                   | -                      | 1.172                         | -                   |
|            | $x_0$                     | 157.7                  | 155.3                         | 111.1               | 807.3                  | 819.3                         | 812.3               |
|            | SSE                       | 16027                  | 16230                         | 18514               | 9897                   | 8831                          | 7371                |
|            | MAD                       | 27.52                  | 27.50                         | 28.67               | 29.45                  | 23.60                         | 22.25               |
|            | MAPE                      | 3.10                   | 3.00                          | 4.88                | 1.81                   | 1.75                          | 1.64                |
| TELESTET   | M                         | 1931.1                 | $+\infty$                     | 2327.5              | 1954.3                 | 8410                          | 2266.5              |
|            | b                         | 1.217                  | 1.169                         | 1.221               | 1.226                  | 1.202                         | 1.228               |
|            | a                         | -                      | 1.194                         | -                   | -                      | 1.202                         | -                   |
|            | $x_0$                     | 82.1                   | 98.2                          | 81.1                | 493.0                  | 504.7                         | 499.8               |
|            | SSE                       | 8524                   | 4251                          | 2650                | 2925                   | 2458                          | 2294                |
|            | MAD                       | 19.82                  | 14.23                         | 17.17               | 11.24                  | 11.45                         | 11.30               |
|            | MAPE                      | 5.25                   | 1.61                          | 5.01                | 1.09                   | 1.00                          | 0.99                |
| COSMOTE    | M                         | 3042.3                 | 2040.3<br>( $\rightarrow 0$ ) | 2545.5              | 3028.0                 | 2570.2<br>( $\rightarrow 0$ ) | 2545.5              |
|            | b                         | 1.517                  | 1.637                         | 1.430               | 1.518                  | 1.655                         | 1.430               |
|            | a                         | -                      | 1.573                         | -                   | -                      | 1.553                         | -                   |
|            | $x_0$                     | 159.5                  | 143.4                         | 158.4               | 159.1                  | 150.3                         | 158.4               |
|            | SSE                       | 15060                  | 17738                         | 17556               | 14945                  | 14168                         | 17556               |
|            | MAD                       | 23.35                  | 28.56                         | 29.44               | 23.44                  | 26.19                         | 29.44               |
|            | MAPE                      | 2.86                   | 2.74                          | 2.97                | 2.85                   | 2.61                          | 2.97                |

**Table 2:** The forecasting evaluation of the model variations to Greek mobile market Data set. Results presented by competitor and for four time periods into the future (Mar 2000 to Dec 2000).

|            |                     |        | Univariate model  |                   | ML model          |                   | EML model         |                   |
|------------|---------------------|--------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
|            |                     | Real   | Dec 95-<br>Dec 99 | Jun 98-<br>Dec 99 | Dec 95-<br>Dec 99 | Jun 98-<br>Dec 99 | Dec 95-<br>Dec 99 | Jun 98-<br>Dec 99 |
| All Market | Mar 00              | 4271.2 | 4293.5            | 4318.9            | 4447.6            | 4335.2            | 4335.2            | 4318.1            |
|            | Jun 00              | 4831.9 | 4644.6            | 4683.7            | 4989.1            | 4739.2            | 4691.0            | 4679.0            |
|            | Sep 00              | 5399.0 | 4926.2            | 4974.7            | 5485.2            | 5061.2            | 4956.1            | 4948.8            |
|            | Dec 00              | 5932.0 | 5160.1            | 5211.4            | 5908.7            | 5297.0            | 5158.3            | 5142.0            |
|            | MAPE <sub>1-2</sub> |        | 2.20%             | 2.09%             | 3.69%             | 1.71%             | 2.21%             | 2.13%             |
|            | MAPE <sub>1-4</sub> |        | 6.54%             | 6.05%             | 2.34%             | 5.09%             | 6.42%             | 6.48%             |
|            | MAD <sub>1-2</sub>  |        | 104.80            | 97.94             | 166.78            | 78.36             | 102.46            | 99.91             |
|            | MAD <sub>1-4</sub>  |        | 363.57            | 335.20            | 110.77            | 282.38            | 355.38            | 360.00            |
| Panafon    | Mar 00              | 1773.0 | 1745.4            | 1783.2            | 1822.9            | 1768.6            | 1790.3            | 1773.3            |
|            | Jun 00              | 1901.9 | 1831.1            | 1898.4            | 1956.5            | 1862.2            | 1903.1            | 1873.2            |
|            | Sep 00              | 2056.9 | 1898.8            | 1998.2            | 2069.5            | 1932.1            | 1999.7            | 1954.0            |
|            | Dec 00              | 2226.0 | 1950.8            | 2082.5            | 2159.6            | 1980.7            | 2083.1            | 2018.8            |
|            | MAPE <sub>1-2</sub> |        | 2.64%             | 0.38%             | 2.84%             | 1.17%             | 0.52%             | 0.76%             |
|            | MAPE <sub>1-4</sub> |        | 6.33%             | 2.52%             | 2.32%             | 4.86%             | 2.56%             | 3.96%             |
|            | MAD <sub>1-2</sub>  |        | 49.20             | 6.89              | 52.21             | 22.07             | 9.19              | 14.49             |
|            | MAD <sub>1-4</sub>  |        | 132.95            | 54.00             | 45.84             | 103.58            | 54.62             | 84.79             |
| Teletet    | Mar 00              | 1278.0 | 1329.0            | 1316.7            | 1264.1            | 1262.7            | 1350.6            | 1310.9            |
|            | Jun 00              | 1414.8 | 1481.8            | 1453.6            | 1362.3            | 1343.1            | 1531.6            | 1441.4            |
|            | Sep 00              | 1580.0 | 1637.9            | 1587.0            | 1445.6            | 1403.6            | 1730.3            | 1571.4            |
|            | Dec 00              | 1645.0 | 1794.3            | 1713.8            | 1512.3            | 1446.1            | 1948.0            | 1704.8            |
|            | MAPE <sub>1-2</sub> |        | 4.35%             | 2.87%             | 2.41%             | 3.15%             | 6.95%             | 2.21%             |
|            | MAPE <sub>1-4</sub> |        | 5.35%             | 2.59%             | 5.36%             | 7.39%             | 10.45%            | 2.15%             |
|            | MAD <sub>1-2</sub>  |        | 58.88             | 38.57             | 33.38             | 43.68             | 94.52             | 29.55             |
|            | MAD <sub>1-4</sub>  |        | 81.12             | 38.13             | 83.57             | 115.79            | 160.48            | 31.80             |
| Cosmote    | Mar 00              | 1220.2 | 1219.1            | 1219.0            | 1360.6            | 1303.9            | 1194.3            | 1233.9            |
|            | Jun 00              | 1515.2 | 1331.7            | 1331.7            | 1670.3            | 1533.9            | 1256.3            | 1364.4            |
|            | Sep 00              | 1762.1 | 1389.5            | 1389.5            | 1970.1            | 1725.5            | 1226.1            | 1423.4            |
|            | Dec 00              | 2061.0 | 1415.0            | 1415.0            | 2236.7            | 1870.2            | 1127.2            | 1418.4            |
|            | MAPE <sub>1-2</sub> |        | 6.11%             | 6.11%             | 10.87%            | 4.05%             | 9.60%             | 5.54%             |
|            | MAPE <sub>1-4</sub> |        | 16.18%            | 16.18%            | 10.52%            | 4.86%             | 23.73%            | 15.37%            |
|            | MAD <sub>1-2</sub>  |        | 92.44             | 92.44             | 147.77            | 51.24             | 142.41            | 82.28             |
|            | MAD <sub>1-4</sub>  |        | 300.96            | 300.96            | 169.82            | 82.48             | 438.65            | 286.47            |

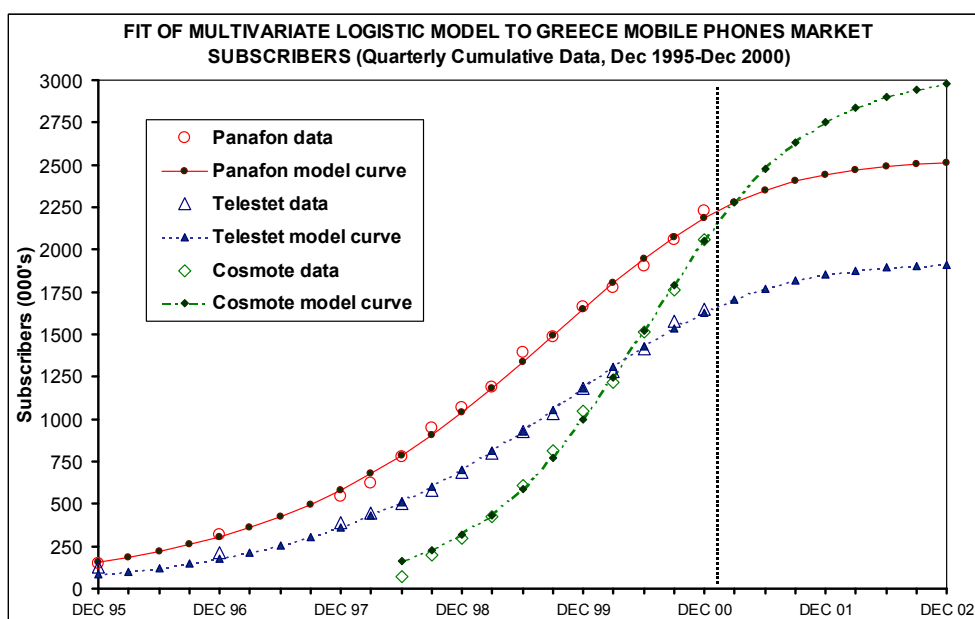


**Figure 1:** Application of the proposed Multivariate Logistic (ML) model to all data of Greek Mobile Phones Market.

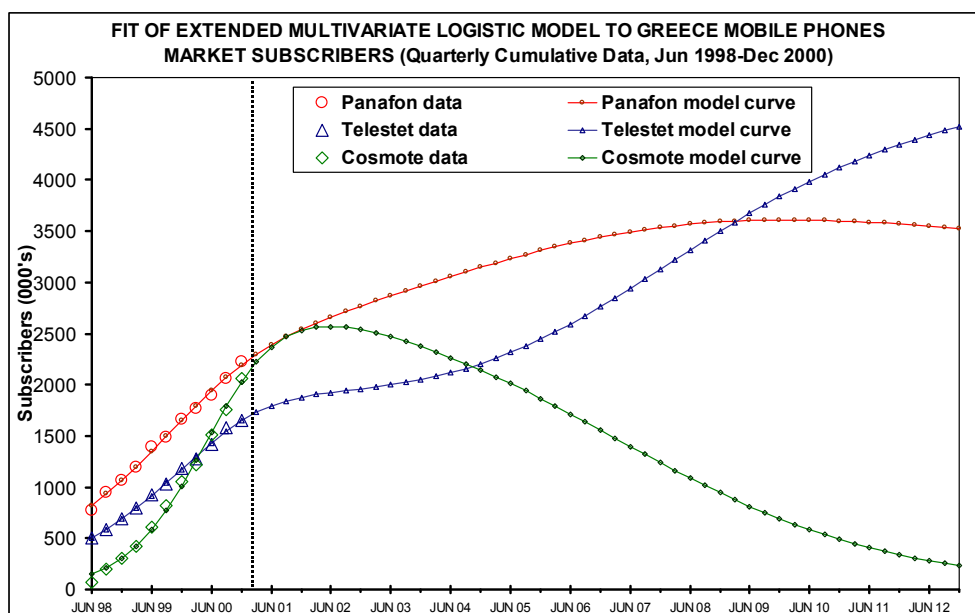


**Figure 2:** Application of the proposed Extended Multivariate Logistic (EML) model to last part of available data of Greek Mobile Phones Market Subscribers.