

ANALYSIS AND FORECASTING OF THE NUMBER OF TRACTORS IN ITALY WITH TIME–SERIES METHODS

Dr. Petros Christodoulou

Associate Professor

Department of Business Administration, University of Macedonia

156, Egnatia Str., GR-540 06, Thessaloniki, Greece

e-mail: christod@uom.gr

Dr. Michail Pazarskis

Adjunct Professor

Department of Accounting, Technological Educational Institute of Serres

Terma Magnisias Str., GR-621 24, Serres, Greece

e-mail: pazarskis@gmail.com

Abstract

In this paper we analyze and forecast the number of tractors in Italy. The particular method consists of 4 stages: identification, estimation, diagnostic checking and forecasting. It proves that the most suitable model is the General Modified Exponential (G.M.E.) with 1st order autocorrelation correction. So we expect that the forecasts of this model will be the best.

Περίληψη

Στην εργασία αυτή χρησιμοποιείται μία μέθοδος χρονοσειρών για την ανάλυση και πρόβλεψη του αριθμού των τρακτέρ της Ιταλίας. Η συγκεκριμένη μέθοδος αποτελείται από 4 στάδια: αναγνώριση, εκτίμηση, διαγνωστικός έλεγχος και σύγκριση των προβλέψεων. Συμπερασματικά προκύπτει ότι η Γενική Τροποποιημένη Εκθετική (Γ.Τ.Ε.) έχει την καλύτερη προσαρμογή στα δεδομένα, άρα αναμένεται να δώσει και τις ακριβέστερες προβλέψεις.

Key words: time series; forecasting; number of tractors.

1. Introduction

Mar - Molinero (1980) used the Logistic curve to model the number (stock) of agricultural tractors in Spain and introduced an autoregressive error term to model the effect of economic influences and government policies. They followed many other papers with different methods with the same data.

In this paper we follow the same process for the number of tractors in Italy. First we present the General Modified Exponential (G.M.E.) and its special cases i.e. the Logistic, the Gompertz and the Simple Modified Exponential. Also the close related Bass model. Then we describe the identification methods, the estimation method, the model parameters and various measures of fit like the R^2 , the S.S.R. and the Durbin – Watson statistic. Next we test the residuals for autocorrelation and correct the models. Finally we take the forecasts from these models and measure their accuracy with the usual measures and the model identification method of Gregg et al. (1964). Then we review the estimation, model checking and forecasts produced from these models. The data – 43 obs. (1961-2003) were taken from the data basis: <http://devdata.worldbank.org/> (All the calculations and diagrams were made with the use of the statistical - econometric package E-views 4.1).

2. The General Modified Exponential (G.M.E.)

The General Modified Exponential (generalized Logistic), Gregg et al. (1964), is given by:

$$y_t = a / (1 + \varphi \times \exp(-b(t-m)))^{1/\varphi} \quad (1)$$

where a = upper asymptote, b = slope parameter, $t = m$ = point of inflexion and φ is α power low parameter.

Special Cases

i) $\phi = 1$ Logistic $y_t = a / (1 + \exp(-b(t-m)))$ (2)

ii) $\phi = 0$ Gompertz $y_t = a \times \exp(-\exp(-b)(t-m))$ (3)

iii) $\phi = -1$ Simple Modified Exponential $y_t = a (1 - \exp(t-m))$ (4)

were $t = 1, 2, \dots, T = \text{time}$.

Note that the Bass model: $y_t = (a - b \times \exp(-ct)) / (1 + d \times \exp(-ct))$ (5)

when $b = 0$ gives the Logistic curve, while when $d = 0$ we have the Simple Modified Exponential.

3. Identification methods

The more elementary methods are the examination of the diagrams of the data, their 1st, 2nd, etc differences, the transformed data (e.g. their logs) and their differences etc. Among the more accurate methods is the method of Gregg et al. (1964) which we describe shortly next:

Greggs method.

A) Calculation of the Moving Average (MA) and the Slope(S) from:

$$MA = (\text{Moving Average at year } t) = (y_{t-2} + y_{t-1} + y_t + y_{t+1} + y_{t+2})/5 \quad (6)$$

$$S = (\text{Slope at year } t) = (-2 \times y_{t-2} - y_{t-1} + y_{t+1} + 2 \times y_{t+2})/10 \quad (7)$$

B) Examination of the scatter plots:

Compute and plot against time	If the result oscillates about a straight line which is:	Then the curve suggested is:
Slope	Horizontal	Straight line
Slope	At an angle to horizontal	Parabola
Slope/(Moving Average)	Horizontal	Simple Exponential
Slope/(Moving Average) ²	At an angle to horizontal	Logarithmic Parabola
Logarithm of Slope	Sloping down to the right	Simple Modified Exponential
Logarithm of {Slope/(Moving Average)}	Sloping down to the right	Gompertz
Logarithm of {Slope/(Moving Average) ² }	Sloping down to the right	Logistic
Log(S)-(φ-1)/φ×log(MA)	Sloping down to the right	GME

where $(\phi-1)/\phi$ is estimated from the regression: $\log(S) = a + b \times t + ((\phi-1)/\phi) \times \log(MA)$

4. Model estimation

The model parameters were estimated with the non linear least squares algorithm, which requires initial parameter values. The estimated parameters of the various models are shown in Table 1 together with the usual econometric measures of fit i.e. R^2 , SSR, and the Durbin – Watson statistic.

Table 1. Estimated parameters of various models

model	a	b	c	d	f	R ²	SSR	DW
Logistic	1778,50	0,1010	16,5000	-	-	0,9985	12860,270	0,6314
Gompertz	1982,60	0,0600	12,0000	-	-	0,9989	9683,1410	0,8109
GME	1895,00	0,3067	-	14,0579	0,0728	0,9990	8620,8420	0,9078
Bass	1857,6	799,33	0,08322	3,33267	-	0,9990	8388,7880	0,9275

From the inspection of this Table we notice the presence of high 1st order autocorrelation in the residuals, so we move to the next stage.

5. Diagnostic checking

In this stage we test the fitted models for autocorrelation in the residuals and re-estimate them. The results are shown in the next table:

Table 2. Estimated parameters of various models (with 1st order autocorrelation correction)

model	a	b	c	d	f	AR(1)	R ²	SSR
Logistic	1800,90	0,097	16,73	-	-	0,654	0,999195	6317,2
Gompertz	1972,09	0,061	12,40	-	-	0,6	0,999201	6271,4
GME	1876,33	0,077	14,61	-	0,42	0,54	0,999238	5980,4
Bass	1853,26	0,084	-	(d ₁ =743,84)	(d ₄ =3,4476)	0,54	0,999240	5965,4

6. Forecasting

The forecasting accuracy of the models is given by various statistical measures like those described below in Tables 3 and 4.

Table 3.

model	Logistic	Gompertz	GME	Bass
RMSE	17,29	15,01	14,16	13,97
MAE	14,36	12,89	11,85	11,41
MAPE	1,84	1,27	1,23	1,11
Theil In. Cr.	0,0074	0,0065	0,0061	0,0060
Bias Prop.	0,0011	0,0001	0,0000	0,0000
Var. Prop.	0,0101	0,0001	0,0005	0,00025
Cov. Pr.	0,9888	0,9988	0,9996	0,9998

Table 4.

model	Logistic	Gompertz	GME	Bass
RMSE	14,82	15,14	14,04	14,07
MAE	11,57	13,02	11,49	11,47
MAPE	1,18	1,27	1,12	1,09
Theil In. Cr.	0,0063	0,0065	0,006	0,006
Bias Prop.	0,0019	0,0009	0,0000	0,0000
Var. Prop.	0,0012	0,0004	0,0004	0,0002
Cov. Pr.	0,9968	0,9987	0,9996	0,9998

From their inspection we conclude again that the G.M.E with 1st order autocorrelation correction has the best fitting to the data.

7. Conclusions

In this paper we use a time series method for the analysis and forecasting of the number of tractors in Italy. The particular method consists of 4 stages: identification, estimation, diagnostic checking and comparison of forecasts. In conclusion it follows that the General Modified Exponential (G.M.E) – with 1st order autocorrelation correction - has the best fitting to the data, so we expect that its forecasts are also the best.

References

1. Gregg, J.V., Hossell, C.H. and Richardson, J.T., (1964) Mathematical Trend Curves: an Aid to Forecasting // I.C.I. Monograph No 1, Oliver and Boyd, Edinburgh.
2. Lilien D.M. et al. (2002) E-views (Econometric Views), Version 4.1 // Quantitative Micro Software, Los Angeles.
3. Mar – Molinero, C., (1980) Tractors in Spain: a Logistic Analysis // Journal of Operational Res. Soc. Vol. 31, 141-152.
4. Meade, N., (1984) The Use of Growth Curves in Forecasting Market Development - a Review and Appraisal // Journal of Forecasting. Vol. 3, 429-451.

Appendix

Figure 1: Original data

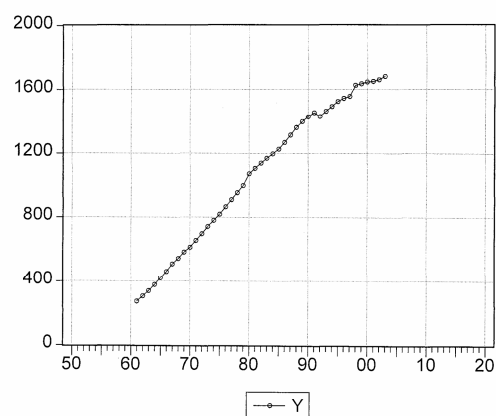


Figure 2: Identification test-1

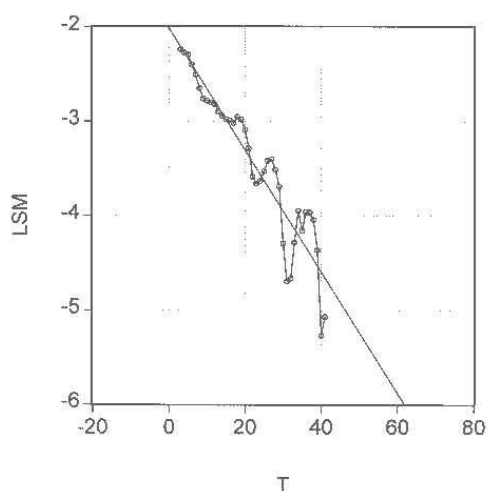


Figure 3: Identification test-2

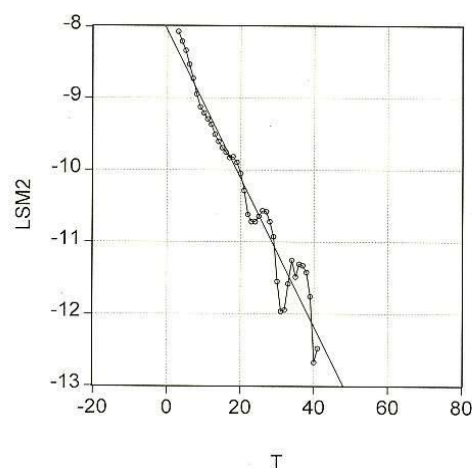


Figure 4: Correlogram of Y

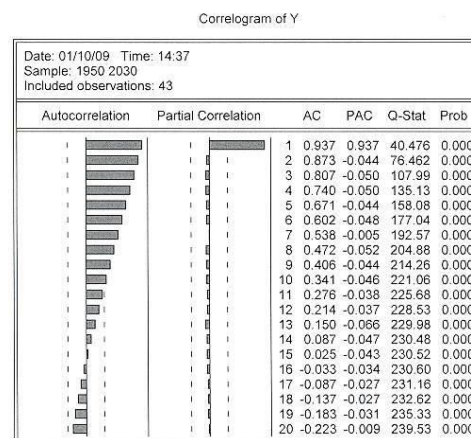


Figure 5: Correlogram of $D(Y)$

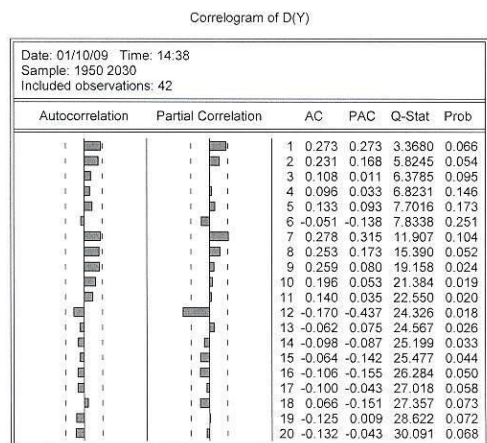


Figure 7: Forecasting the models with autocorrelation correction

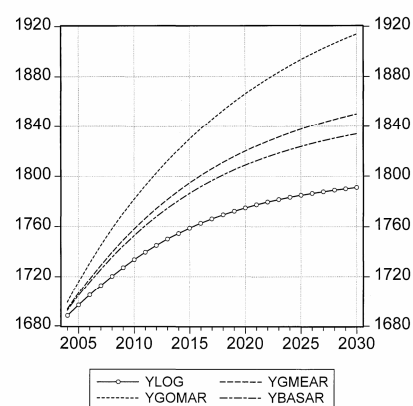


Figure 6: Forecasting the models

